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by

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Abstract

In this paper we use the put-call parity to calculate the premium for early exercise of put options on the DAX index. Because this is a performance index, it is not necessary to separate this premium from the early exercise premium of a call option. We find the early exercise premium of a put option to be positively correlated with the moneyness and the standard deviation of the returns on the index.

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PUT-CALL PARITIES AND THE VALUE OF EARLY EXERCISE FOR PUT OPTIONS ON A PERFORMANCE INDEX

Abstract

In this paper we use the put-call parity to calculate the premium for early exercise of put options on the DAX index. Because this is a performance index, it is not necessary to separate this premium from the early exercise premium of a call option. We find the early exercise premium of a put option to be positively correlated with the moneyness and the standard deviation of the returns on the index.

1. Introduction

In an interesting study Zivney (1991) demonstrates that deviations from the European put-call parity are caused by the possibility of early exercise. He investigates this possibility for options on the S&P 100 index. Because options on the S&P 100 index are not corrected for dividend payments, both call and put options may contain a premium for early exercise, thereby causing deviations from the European put-call parity. The deviation from the put-call parity is very difficult to split in two parts. This is not the case if options on a performance index are considered. In a performance index it is assumed that dividends are reinvested in the index, which makes the index similar to a non dividend paying stock. As Merton (1973) has shown, rational investors will only early exercise call options just before an ex dividend date, so they will not exercise call options on a performance index before their expiration date. Deviations of the put-call parity for options on a performance index can therefore only be caused by the possibility of early exercise of put options.

As far as we know Zivney's concept has not been applied to options on a performance index. The reason for this is that options on a performance index are hardly available. Fortunately, since May 1993, options on the DAX index are traded on the Amsterdam Stock Exchange (ASE)[1]. The DAX index is an example of a performance index [see e.g. Grünbichler and Callahan (1994)]. This therefore gives us a good opportunity to investigate the value of early exercise for American put options. Knowledge about this premium is helpful in developing an option formula for American put options.

The remainder of this paper is organized as follows. In section 2 put-call parities will be discussed and we will derive a new parity relation for American options on an index with a known dividend yield. In this section it will also be discussed what can be expected when we model the early exercise premium for put options on the DAX. In section 3 the data

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[1] We notice that these securities were issued under the name index *warrants*. However, they are in fact index options, except that they are traded on the stock exchange, instead of the options exchange. Because this only involves some institutional differences which are not important for valuation purposes, we will simply refer to these index warrants as index options.

and the regression analysis, which will be used when investigating the premium for early exercise, will be discussed. In section 4 the results will be presented and discussed. Finally section 5 will give a summary and some conclusions.

2. Put-call parities and the value of early exercise

Depending upon the assumptions that are made about the possibility of early exercise and about dividend payments different put-call parities - barring arbitrage possibilities - can be derived. Assuming for instance that early exercise is not possible and that a continuous and known dividend is paid on an index, the following European put-call parity can be derived (Chance, 1987):

$$\hat{C}_t = P_t + I_t e^{-q(T-t)} - X e^{-r(T-t)} \tag{1}$$

In which:

P_t	=	the market value of the put option at time t,
I_t	=	the market value of the index at time t,
q	=	the known annualized continuous dividend yield of the index,
X	=	the exercise price of the option,
T	=	the expiration date of the option,
r	=	the known annualized continuous interest rate until the time to the expiration date (T-t), and
$\hat{C}$	=	the value of the call option according to the parity relation.

If the assumption is made that early exercise is possible then it is no longer possible to derive the put-call parity in the form of an equality. Instead it is only possible to give a lower and an upper bound for the value of the call option in terms of the other variables. If the assumption that the index has a known dividend yield is still made and in addition it is assumed that the interest rate is known and equal for all maturities until T, then the lower and upper bounds for the value of the call options become:

$$\hat{C}_t^{lower} \geq P_t + I_t e^{-q(T-t)} - X \tag{2}$$

$$\hat{C}_t^{upper} \leq P_t + I_t - Xe^{-r(T-t)}. \tag{3}$$

The derivation of these bounds is presented in the appendix[2].

In case no dividends are paid on the index (i.e. q=0) it is clear that the upper bound for the American call option, equation (3), is the same as the value of the call option according to the European put-call parity, equation (1). The difference between the upper and lower bound for the American call option can then be interpreted as the maximum premium that will be paid on the put option, since the value of an American call option on a no dividend paying index will be equal to the value of its European counterpart.

The actual premium for early exercise will be equal to the actual difference between the value of the call option and the European put-call parity. Zivney (1991) investigates the premium for early exercise in a similar way, but since he uses options on the S&P 100, which is a dividend paying index, he has to make the additional assumption that the in-the-money option will have a larger premium for early exercise than the out-of-the-money option. Since we use options on a non dividend paying index we do not have to make that assumption. Moreover we know that the difference between the actual value of the American call option on the index and the value according to the European put-call parity will be solely determined by the premium for early exercise of the American put option.

The investor has an option to exercise early. Zivney (1991) investigates whether the premium for early exercise depends on (1) the moneyness; (2) the interest rate and (3) the time to maturity. Since we will only calculate the value of early exercise for put options, we will restrict ourselves to explaining his hypotheses for put options. Since the probability

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[2] Still other put-call parities can be derived if it is assumed for instance that discrete dividend payments are made. The parities that result in this case can be found in Jarrow & Rudd (1983).

of early exercise of an American put option will be larger when the option is deeper in-the-money (Merton, 1973), it may be expected that the premium for early exercise will increase when the value of the index decreases for any given exercise price. If the risk free interest rate increases then the present value of the exercise value decreases (see e.g. Geske and Johnson, 1984), thereby making the possibility of early exercise more attractive. The risk free interest rate therefore is expected to have a positive influence on the premium for early exercise. Finally, the time to expiration is also expected to have a positive effect on the early exercise premium. This is easy to see when we consider two American put options which differ only with respect to their expiration dates, T_1 and T_2, where $T_2 > T_1$. Since the option with the longer maturity (T_2-t) has the same possibilities as the option with the shorter maturity (T_1-t) plus all the options associated with the longer maturity (T_2-T_1), its value can never be smaller than the value of the option with the shorter maturity. In his study Zivney (1991) indeed finds the positive signs he a priori expected. In addition to the factors investigated by Zivney (1991) it is also possible to study the effect of the volatility. This effect is also expected to be positive, since a higher volatility implies that the probability for the put option to become in-the-money increases, thereby increasing the value of early exercise.

3. Data description and methodology

We use closing prices for options on the DAX index, traded on the ASE, for the period of May 12, 1992, when they were first traded, until October 7, 1993. These closing prices are taken from Datastream. Since we only want to use dates for which both the call and the put options were traded, this results in a total of 175 observations. Closing values for the DAX index, closing values for the German Mark/Dutch Guilder exchange rate and estimates of the risk free interest rate were also taken from Datastream. For the risk free interest rate we used the yield on German treasury bills with a maturity closest to the maturity of the options. Data for the exchange rate are needed because the options are traded in Dutch Guilders, while the DAX index and the exercise price are listed in German Marks. Wei (1992) shows that the value of the option traded in Dutch Guilders will be equal to the value of a similar option traded in German Marks, multiplied by the current exchange rate.

The behavior of the premium for early exercise with respect to the variables that were discussed in section 2 will be investigated by means of the following multiple regression equation:

$$\begin{aligned} D_t = \beta_0 + \beta_1 * M_t + \beta_2 * \sigma_{impl,t-1} + \beta_3 * r_t + \\ + \beta_4 * (T-t) + \beta_5 * D_{t-1} + \varepsilon_t . \end{aligned} \tag{4}$$

The dependent variable D_t is defined as:

$$D_t = \frac{\hat{C}_t - C_t}{\hat{C}_t}. \tag{5}$$

As discussed in section 2 the premium for early exercise is equal to $\hat{C}_t$-C_t, the difference between the value of the call option according to the parity model and the actual value of the call option, which is also equal to the difference between the American put value and the European put value. We use the relative difference in terms of the model price as the dependent variable to correct for heteroskedasticity problems. The regressor M_t measures the relative amount by which the put option is in-the-money:

$$M_t = \frac{X - I_t}{X}. \tag{6}$$

As discussed in section 2 this variable is expected to have a positive effect on D_t. The volatility is measured by the implied standard deviation of the call option, $\sigma_{impl,t-1}$, according to the Black & Scholes (1973) formula. Since Black & Scholes (1973) assume that early exercise is not possible and since an American call option on the DAX behaves like its European counterpart it seems appropriate to use the implied standard deviation of the call option rather than the put option. The implied standard deviation at time t-1 rather than time t is used, since σ_t is a function of the dependent variable and will probably not be uncorrelated with the error term ϵ_t. As also discussed in section 2 the interest rate, r_t, and the time to maturity, (T-t), are expected to have a positive effect on the premium and thus on D_t. To correct for autocorrelation problems D_{t-1} is also used as a regressor.

4. Results and discussion

In figure 1 we have included the lower and upper bounds of the put-call parity (equations (2) and (3)) as well as the actual call prices.

[Insert Figure 1]

As can be seen in this figure the call price is almost always between the upper and lower bound. It turns out that the call price is above the upper bound on only two days. These observations are omitted in the analysis that follows.

Summary statistics for the variables, the premium and the premium as a fraction of the value of the call option according to the European put-call parity are included in table 1.

[Insert Table 1]

As table 1 shows the premium for early exercise is always positive with an average of 0.616 Dutch guilders, which corresponds with $D_t = 30.3\%$.

Estimation of equation (4) yields the following result (t-values in parentheses):

$$D_t = \underset{(-1.54)}{-0.09} + \underset{(7.54)}{0.99}\, M_t + \underset{(4.10)}{0.54}\, \sigma_{impl,t-1} + \underset{(1.56)}{1.71}\, r_t - \underset{(-1.00)}{0.03}\, (T-t) + \underset{(9.44)}{0.54}\, D_{t-1} + \varepsilon_t\,, \tag{7}$$

with: $\hat{R}^2 = 0.9293$.

The value of the Durbin's h-statistic is 1.904, implying that there is no autocorrelation. The value of $\hat{R}^2$ shows that the selected variables do a good job in explaining the value of early exercise. As expected the signs for M_t and $\sigma_{impl,t-1}$ are both positive, indicating that the early exercise premium indeed behaves like an option with respect to these variables. The size of the coefficient for M_t shows that this variable is not only statistically but also economically significant. If the value of the index changes with e.g. 1% in terms of the exercise price, the premium changes (ceteris paribus) by 0.99% in terms of the upper bound of the call price. Changes of 1% in M_t are of course not uncommon. The coefficient for $\sigma_{impl,t-1}$ is also economically significant, since each percentage point change in the volatility translates (ceteris paribus) in a 0.54% change in the premium. Changes of 1%

point in the volatility are not unlikely given the standard deviation of the volatility of 0.73% (see table 1). The positive value of D_{t-1} shows the positive autocorrelation, implying that a high premium at time t will probably be followed by a high premium at time t+1. The coefficient of r_t is positive as expected, but not statistically significant. Since the interest rate typically doesn't change very much from day to day, there's no economic significance either. Finally, the negative sign of the time to maturity is contrary to what might be expected. For this variable there's a lack of both statistical and economical significance however.

5. Summary and conclusions

In this paper we have calculated the value of early exercise of a put option from deviations of the European put-call parity. Because we have considered put options on a performance index, it turned out to be possible to isolate this premium. As hypothesized by us, the premium for early exercise was significantly and positively correlated with the moneyness and the volatility of the index. The relationship of the premium with the interest rate is positive, as expected, but not significant. Contrary to our expectations we find a negative relation with the time to maturity, however this relation is not significant. The results of this study may be particularly useful for the development of a correct model for the pricing of American put options.

References

Black, F. and M. Scholes, 1973, The Pricing of Options and Corporate Liabilities, Journal of Political Economy 81, 637-654.

Chance, D.M., 1987, Parity Tests of Index Options, Advances in Futures and Options Research 2, 47-64.

Geske, R. and H.E. Johnson, 1984, The American Put Valued Analytically, The Journal of

Finance 39, 1511-1524.

Grünbichler, A. and T.W. Callahan, 1994, Stock Index Futures Arbitrage in Germany: The Behavior of the DAX Index Futures Prices, Review of Futures Markets (forthcoming).

Jarrow, R.A. and A. Rudd, 1983, Option pricing (Richard D. Irwin, Homewood, Illinois).

Merton, R.C., 1973, Theory of Rational Option Pricing, Bell Journal of Economics and Management Science 4, 141-183.

Wei, J.Z., 1992, Pricing Nikkei Put Warrants, Journal of Multinational Financial Management 2, 45-75.

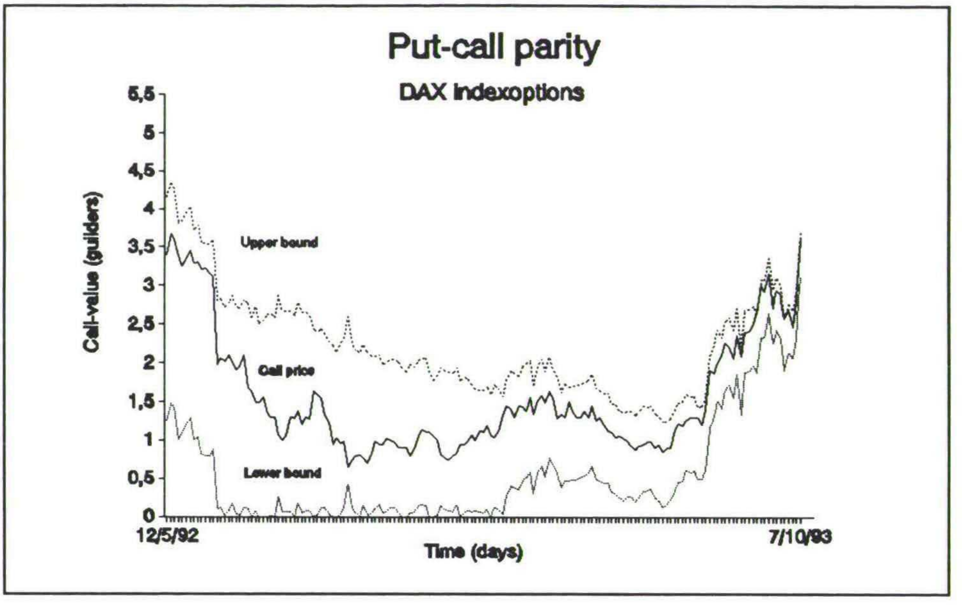
Zivney, T.L., 1991, The Value of Early Exercise in Option Prices: An Empirical Investigation, Journal of Financial and Quantitative Analysis 26, 129-138.

Table 1: Summary statistics for the early exercise premium

				quantiles			
	average	std.dev.	smallest	5%	50%	95%	largest
$\hat{C}_t$-C_t (in guilders)	0.616	0.171	0.067	0.133	0.535	1.404	1.964
$D_t=(\hat{C}_t-C_t)/\hat{C}_t$	30.3%	6.82%	2.4%	4.7%	27.2%	60.7%	75.4%
$M_t=(X-I_t)/X$	4.2%	2.65%	-15.2%	-10.5%	5.5%	14.8%	17.7%
$\sigma_{impl,t-1}$	17.5%	0.73%	13.7%	14.6%	16.3%	25.1%	29.3%
r_t	6.3%	0.40%	5.9%	5.9%	7.1%	8.9%	8.9%
T-t (in years)	1.2	0.42	0.51	0.61	1.26	1.85	1.89

Note: standard deviations are corrected for autocorrelation

Figure 1: Upper and Lower bounds of the put-call parity for options on the DAX index for May 12, 1992 to October 7, 1993.



Appendix: Derivation of the American put-call parity

Both equation (2) and (3) can be derived by constructing a portfolio which must always have a positive value in order to exclude arbitrage possibilities. For the derivation of the lower bound, equation (2), we need to construct a portfolio by taking the following positions:

1. A short position in the underlying index of $I_t e^{-q(T-t)}$;
2. A short position in a put option written on the index;
3. A long position in a call option written on the index with the same exercise price (X) and the same expiration date (T) as the put option;
4. A long position in a risk free bond with value X.

It is assumed that if dividends have to be paid because of the short position in the index, additional units of the index are sold short by the amount of the dividend obligation.
Exercise is possible at the expiration date T, or at a time τ for which: $t<\tau<T$. For ease of exposition we will only consider the possibility of early exercise of the written put option, since early exercise of the call option can never reduce value for a rational investor. Thus, early exercise will only take place if the put option is in-the-money, i.e.: $X \geq I_\tau$.
The pay off for the investor in case of early exercise is given in table 2.

Value at time t		**Exercise at τ**	**Exercise at T**	
		$X \geq I_\tau$	$X \geq I_T$	$X < I_T$
short index	$-I_t e^{-q(T-t)}$	$-I_\tau e^{-q(T-\tau)}$	$-I_T$	$-I_T$
short put	$-P_t$	$I_\tau - X$	$I_T - X$	0
long call	C_t	0	0	$I_T - X$
long bond	X	$Xe^{r(\tau-t)}$	$Xe^{r(T-t)}$	$Xe^{r(T-t)}$
	Total Pay off:	$I_\tau(1-e^{-q(T-\tau)}) + X(e^{r(\tau-t)}-1) \geq 0$	$X(e^{r(T-t)}-1) \geq 0$	$X(e^{r(T-t)}-1) \geq 0$

Table 2: The pay off for the investor when exercised.

Since the value at exercise is positive at any moment, the value of the portfolio must also be positive at time t, resulting in equation (2).

For the derivation of equation (3) we need a slightly different portfolio consisting of:

1. A long position in the index of I_t;

2. A long position in a put option written on the index;

3. A short position in a call option written on the index with the same exercise price, X and the same expiration date, T, as the put option;

4. A short position in a risk free bond with value $Xe^{-r(T-t)}$.

If we now assume that the dividends which are received on the index are continuously reinvested in the index, equation (3) can be derived in a similar way as equation (2).

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